3/6/17 "Dont cry because its over, smile because it happened."-Dr. Seuss

HW: "Sequences" homework section

AIM: What are Sequences?

Warm Up:

(pattern)

Sequences are extremely important in mathematics, both theoretical and applied. A **sequence** is formally defined as **a function that has as its domain the set the set of positive integers**, i.e. $\{1, 2, 3, \ldots, n\}$.

***Exercise* #1:** A sequence is defined by the equation $a(n) = 2n - 1$.

(a) Find the first three terms of this sequence, denoted by a_1, a_2, and a_3.

$a_1 = 2(1) - 1 = 1$

$a_2 = 2(2) - 1 = 3$

$a_3 = 2(3) - 1 = 5$

(b) Which term has a value of 53?

$53 = 2n - 1$
$+1 \quad\quad +1$

$\frac{54}{2} = \frac{2n}{2}$

$27 = n$

53 is the 27^{th} # in the sequence.

(c) Explain why there will not be a term that has a value of 70.

$70 = 2n - 1$
$+1 \quad\quad +1$

$\frac{71}{2} = \frac{2n}{2}$

$35.5 = n$

Not an integer so 70 is not in the sequence

* Sequences are patterns.

Recall that sequences can also be described by using **recursive definitions**. When a sequence is defined recursively, terms are found by operations on previous terms.

Recursive: you need to use the previous term(s)

previous term

***Exercise* #2:** A sequence is defined by the recursive formula: $f(n) = f(n-1) + 5$ with $f(1) = -2$.

(a) Generate the first five terms of this sequence. Label each term with proper **function** notation.

$f(1) = \boxed{-2}$

$f(2) = -2 + 5 = \boxed{3}$

$f(3) = 3 + 5 = \boxed{8}$

$f(4) = 8 + 5 = \boxed{13}$

$f(5) = 13 + 5 = \boxed{18}$

(b) Determine the value of $\boldsymbol{f(20)}$. Hint – think about how many times you have added 5 to **−2**.

$f(20) = -2 + 19(5)$

$f(20) = 93$

***Exercise* #3:** Determine a recursive definition, in terms of $f(n)$, for the sequence shown below. Be sure to include a starting value.

5, 10, 20, 40, 80, 160, …

$f(1) = 5$

$f(n) = f(n-1) \cdot 2$

"Previous term times 2"

***Exercise* #4:** For the recursively defined sequence $t_n = (t_{n-1})^2 + 2$ and $t_1 = 2$, the value of t_4 is

(1) 18 (3) 456

(2) 38 (4) 1446

"square previous term and then add 2"

$t_1 = 2$

$t_2 = 2^2 + 2 = 6$

$t_3 = 6^2 + 2 = 38$

$t_4 = 38^2 + 2 = 1446$

***Exercise* #5:** One of the most well-known sequences is the Fibonacci, which is defined recursively using two previous terms. Its definition is given below.

term right before

2 terms before

$$f(n) = f(n-1) + f(n-2) \text{ and } f(1) = 1 \text{ and } f(2) = 1$$

Generate values for $f(3)$, $f(4)$, $f(5)$, and $f(6)$ (in other words, then next four terms of this sequence).

$f(1) = 1$
$f(2) = 1$
$f(3) = 1 + 1 = 2$
$f(4) = 1 + 2 = 3$
$f(5) = 2 + 3 = 5$

$f(6) = 3 + 5 = 8$

$f(3) = 2$
$f(4) = 3$
$f(5) = 5$
$f(6) = 8$

It is often possible to find algebraic formulas for simple sequences, and this skill should be practiced.

***Exercise* #6:** Find an algebraic formula $a(n)$, similar to that in *Exercise* #1, for each of the following sequences. Recall that the domain that you map from will be the set $\{1, 2, 3, \ldots, n\}$.

(a) 4, 5, 6, 7, ...

n

$4 = 1 + 3$
$5 = 2 + 3$
$6 = 3 + 3$
$7 = 4 + 3$

$a(n) = n + 3$

(b) 2, 4, 8, 16, ...

$2^1 \quad 2^2 \quad 2^3 \quad 2^4$

$\boxed{a(n) = 2^n}$

(c) $\frac{5}{1}, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \ldots$

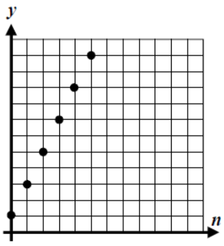
$a(n) = \frac{5}{n}$

(d) $-1, 1, -1, 1, \ldots$

(e) 10, 15, 20, 25, ...

(f) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

***Exercise* #7:** Which of the following would represent the graph of the sequence $a_n = 2n + 1$? Explain your choice.

(1)


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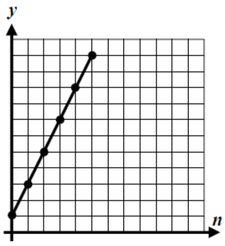

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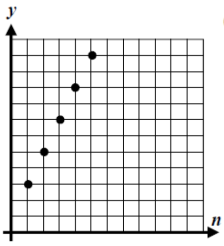

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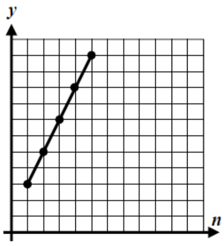

Explanation: